Parallel PEPS Tool Performance Analysis Using Stochastic Automata Networks[⋆]

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Abstract. This paper presents a theoretical performance analysis of a parallel implementation of a tool called Performance Evaluation for Parallel Systems (PEPS). This software tool is used to analyze Stochastic Automata Networks (SAN) models. In its sequential version, the execution time becomes impracticable when analyzing large SAN models. A parallel version of PEPS using distributed memory is proposed and modelled with SAN formalism. After, the sequential PEPS itself is applied to predict the performance of this model.

1 Introduction

In recent years, the effort of many authors has confirmed the importance of performance prediction of parallel implementations. Some authors have presented generic studies offering options to the performance prediction of parallel implementations [1, 2]. The research community classifies the different approaches in three quite distinct groups: monitoring, simulation and analytical modelling. This last approach (analytical modelling), compared to the two first ones, is more rarely employed to achieve parallel programs performance prediction. This happens due to a frequent misconception: the most known formalisms to analytical modelling, *e.g.*, Markov chains [3] and queueing networks [4], are not very suitable to represent parallelism and synchronization. In this paper, we adopted the analytical modelling approach using a formalism called Stochastic Automata Networks (SAN). The reader interested in a formal description of the formalism can consult previous publications [5]. The SAN formalism describes a complete system as a collection of subsystems that interact with each other. Each subsystem is described as a stochastic automaton, *i.e.*, an automaton in which the transitions are labelled with probabilistic and timing information. The analysis of the theoretical SAN models is performed by a software package called Performance Evaluation for Parallel Systems (PEPS) [6]. Although PEPS has proven its usability during the past years, it presents an important drawback: the execution time to analyze SAN models with too many states is very often impracticable. Thus, the main contribution of this paper is to

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verify the feasibility of a parallel implementation of the PEPS tool using a SAN model, identifying the requirements and advantages of such approach.

2 PEPS Implementation Analysis

The input of PEPS is a SAN model described in a predefined grammar. The current application loads this grammar and builds an equation system using the events rates. The SAN models allow a better management of the needs for space memory than Markov Chains, because they are described in a more compact and efficient way. This optimization can be carried out due to the use of tensorial algebra [5]. Thus, a model is no more described by a unique matrix, but instead, by a new structure called Markovian Descriptor. This structure is the kernel of the PEPS tool and it has a significant impact over its execution time. Considering a network with N automata and E synchronizing events, the Markovian Descriptor is given by:

$$Q = \bigoplus_{i=1}^{N} Q_l^{(i)} + \sum_{e=1}^{E} \left(\bigotimes_{i=1}^{N} Q_{e^+}^{(i)} + \bigotimes_{i=1}^{N} Q_{e^-}^{(i)} \right) \tag{1}$$

In this equation, there are N matrices $Q_l^{(i)}$ representing the local events and $2E$ matrices $Q_{e_k}^{(i)}$ representing the synchronizing events, which result in a total of $N + 2E$ stored matrices. The basic operation to solve a SAN model in numeric iterative methods (like the Power Method, GMRES, etc.) is to multiply a probability vector π by a Q matrix stored in the Markovian Descriptor form. This probability vector assigns a probability $\pi_i \ (i \in \{1, 2, ..., n\})$ to each one of the n states of the Markov Chain equivalent to the SAN model. Each element of Q may represent a local event or a synchronizing event. The local events are given by a sum of normal factors. On the other hand, synchronizing events are given by a product of normal factors [5]. Hence, the most important point in this descriptor-vector multiplication is to know how one can multiply normal factors. In spite of the Markovian Descriptor optimization, the PEPS application still suffers from a performance decline at the same time that the complexity[3] of the model grows (more details can be found in [6]).

3 PEPS Parallel Version

In order to improve the PEPS software tool performance, this paper proposes a parallel version for this tool based on the features of a specific kind of high performance architecture. To reinforce the usability of this new version, the hardware environment should be based on an usual and not very expensive (compared to a supercomputer) one. Following this scenario, the new version is designed to run over a cluster architecture which has several processors connected by a dedicated fast network.

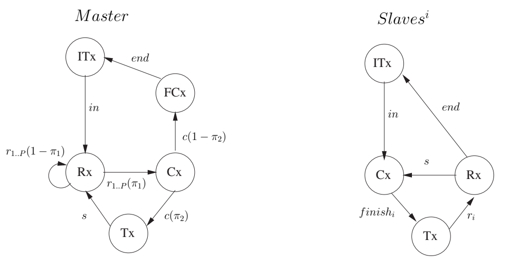
As seen in section 2, PEPS solves a SAN model using numeric iterative methods. In order to represent a SAN model, many matrices are created, describing local and

[3] Complexity here is related to synchronizing events and states amount.

synchronized events rates. Another feature of PEPS is that the number of iterations necessary to make the model converge is different from one input model to another. Due to this, it is not possible to deduce how many iterations are necessary to determinate the convergence of a model. The solution proposed here is based on a synchronized master-slave model. Considering that a SAN model is described as a set of tensorial matrices (from the Markovian Descriptor, equation 1) and each iteration represents the multiplication of a probability vector by these matrices, the main idea is to distribute a set of matrices to each slave and execute the multiplications concurrently. In each iteration, the master node sends, in broadcast, the vector from the i^{th} iteration to the slaves. Each slave takes some time processing its own task, and returns to the master a piece of the next iteration vector. For each iteration, the master must wait for the results from all slaves (reduce operation) to combine them into the next iteration vector. Finally, the master sends this new probability vector in broadcast to the slaves, starting a new iteration. This procedure will continue until the convergence criteria of the numerical method is matched.

4 SAN Model for parallel PEPS

Taking on a parallel point of view, the main relevant states to be modelled using SAN are those who are focused on processes data exchange and computing time. That happens because the trade-off between these two features is the key to determine the success of the parallel implementation. The SAN model which describes the PEPS parallel implementation explained in section 3 is presented in Fig. 1. The model contains one automaton *Master* and *P* automata $Slave^{(i)}$ (*i* = 1..*P*).



Event	Rate
in	α
end	β
c	γ
$r_{1..P}$	δ
$finish_i$	λ

Fig. 1. The PEPS SAN model.

The automaton *Master* has five states *ITx*, *Tx*, *Rx*, *Cx* and *FCx*. It is responsible over the distribution of iteration vectors to the slaves. The states *ITx*, *Tx* and *Rx* mean, respectively, the initial transmission of the matrices and the first vector to the slaves, transmission of new iteration vectors to slaves, and the reception of the resulting vectors evaluated by the slaves. The states *Cx* and *FCx* represent, respectively, the time spent to sum all slaves evaluated vectors and write on file the asked results. The occurrence of the synchronizing event *in* broadcasts the matrices and the first vector to the slaves. On the other hand, the occurrence of the event *end* finalizes the communication between master and slaves. The synchronizing event *s* broadcasts the vector of the i^{th} iteration to slaves. The synchronizing event r_i receives one resulting vector of the slave *i*. The occurrence of events $r_{1..P}$ can change the state of master automaton or not, depending on the probability π_1. The master will change to *Cx* state when the last slave sends its results and goes to *Rx* state, *i.e.*, the master must wait until all slaves send their results of the i^{th} iteration. The *Master* automaton initializes new iterations or finalizes the descriptor-vector multiplication by the occurrence of the local event *c*. In 99% of the times (represented by the probability π_2) the event *c* will initialize new iterations. On the remaining time $(1 - \pi_2)$, the event *c* will finalize the descriptor-vector multiplications.

Automaton $Slave^{(i)}$ represents the slave of index i, where $i = 1..P$. It has four states: *I* (Idle), *Cx* (Computing), *Tx* (Transmitting) and *Rx* (Receiving). All slaves start their tasks when synchronizing event *in* occurs. Slave *i* stops processing a task by the occurrence of the local event $finish_i$. The same slave sends its resulting vector to the master with the synchronizing event r_i. When the event *s* occurs, all slaves start to compute a new step. The occurrence of the event *end* finalizes the execution of the slaves processes.

In order to complete the model representation it is necessary to assign occurrence rates to its events. The rate of one event is the inverse of the time spent in its associated state. The occurrence rate of the event end is extremely high because the time spent to finish a model is insignificant. The occurrence rates of events s, in and c are inversely dependent by the probability vector size and also by the number of slaves nodes. On the other hand the rate of the events $r_{1..p}$ are only inversely dependent by the probability vector size. The rate of the events $finish_{1..p}$ are inversely dependent by the number of multiplications each slave will perform. This number of multiplications is directly dependent by the probability vector size and inversely dependent by the number of slaves nodes. All these rates were obtained through sample programs performed over the target architecture[4], *e.g.*, the time spent by one slave to compute a probability vector was obtained through a program that simulates the number of multiplications performed by one slave).

5 Results

The results given by PEPS are steady state probabilities. For instance, the time that the system stays in state *Tx* of the automaton *Master* is equal to the probability of the system to be in this state. These probabilities are important information to indicate the amount of time the system will stay in a given state. For a parallel implementation this information will help to verify if the proposed parallel solution has bottlenecks which compromise its performance.

Three case studies will be presented next. Each one uses an hypothetical different input SAN model which differs in the number of states (directly related to the size of the probability vector). The behavior of the PEPS parallel version will be analyzed for each input model. Readers must pay attention that the sequential version of PEPS is

[4] The target architecture is a COW (Cluster of Workstations)

Table 3. Results for the case study 16,384.

slaves	Tx_{Master}	Rx_{Master}	Cx_{Master}	Cx_{Slave}
2	0.3690	0.1834	0.0025	0.0585
3	0.3790	0.1887	0.0011	0.0369
4	0.3810	0.1910	0.0005	0.0251
5	0.3811	0.1939	0.0003	0.0177
6	0.3808	0.1961	0.0002	0.0128
7	0.3805	0.1975	0.0001	0.0092

Table 4. Results for the case study 65,536.

slaves	Tx_{Master}	Rx_{Master}	Cx_{Master}	Cx_{Slave}
2	0.3687	0.2258	0.0007	0.0797
3	0.3788	0.2034	0.0002	0.0449
4	0.3808	0.1987	0.0001	0.0289
5	0.3809	0.1984	0.0001	0.0197
6	0.3806	0.1988	3.79e-5	0.0139
7	0.3804	0.1992	2.31e-5	0.0099

Table 5. Results for the case study 327,680.

slaves	Tx_{Master}	Rx_{Master}	Cx_{Master}	Cx_{Slave}
2	0.3628	0.2422	6.95e-5	0.0926
3	0.3764	0.2098	2.45e-5	0.0490
4	0.3796	0.2018	1.14e-5	0.0306
5	0.3802	0.2000	6.18e-6	0.0206
6	0.3802	0.1999	3.58e-6	0.0144
7	0.3802	0.1999	2.18e-6	0.0102

Table 6. Heterogeneous results - 16,384.

$degree$	Cx_{S1}	Cx_{S2}	Cx_{S3}	Cx_{S4}	Cx_{S5}	Cx_{S6}
none	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128
low	0.0120	0.0144	0.0120	0.0144	0.0120	0.0114
medium	0.0161	0.0161	0.0051	0.0161	0.0161	0.0016
high	0.0218	0.0190	0.0087	0.0087	0.0044	0.0044

used to analyze the SAN model that represents the proposed parallel version of PEPS, which needs some input SAN models to solve. Another important remark is that for each new slave added, the SAN model of the PEPS parallel version changes because a new automaton Slave must be added to the model. Thus, the sequential PEPS execution time to solve the parallel PEPS model strongly increases and that is the reason why we have carried out experiments with only until seven slaves nodes for each case study.

Table 3, Tab. 4 and Tab. 5 show the state stay probabilities for each case study. The first important remark is that the probabilities of slave nodes be computing (Cx_{slave}) decline as the number of slaves grows, indicating that more work is done in parallel. Another interesting result can be observed in the probability of master to be receiving data from slaves(Rx_{master}): each case study presents a different behavior. In the 16,384 states case study, this probability increases as the number of slaves grows. That happens because slaves have progressively less work to compute, thus increasing the frequency they send their results to the master. Therefore, for this size of input model, more than two slaves seems to compromise the master efficiency. Looking at the same probability for the others input models (65,536 and 327,680 states) and applying the same analysis, it is possible to identify the best number of slaves (*i.e.*, the point the master becomes the bottleneck of the system) at five for the second case study and probably seven for the third case study. Finally, looking comparatively at the probability of slaves to be computing (Cx_{slave}) in all three case studies, one can see that, as the number of states of the input model grows, the time each slave spends computing gets higher. This is a coherent result, because if the system has a higher workload, it is expected that the nodes spend more time computing.

Until this point, all results presented were based on an homogeneous environment, *i.e.*all slaves have exactly the same computational power and workload (represented by the matrices computational cost). In order to verify the importance of the workload balance for the parallel PEPS implementation, heterogeneity in the slaves behavior was introduced in the SAN model for the parallel version of PEPS. That was done by using different computing rates for each slave. Table 6 presents the results of this experiment, where using the input model with 16,384 states and fixing the number of slaves on 6, four different configurations were analyzed: none, low, medium or high heterogeneity. The results show that as heterogeneity grows, the degree of discrepancy among the slaves to be in state Cx increases. Through this analysis, the model seems to be capable of identify the impact heterogeneity in the slaves behavior could represent over a parallel implementation.

6 Conclusion

The main contribution of this paper is to point out the advantages and problems of the analytical modelling approach applied to predict the performance of a parallel implementation. According to authors best knowledge, some others efforts to predict performance of parallel systems were not so formal. The recent work of Gemund [7] exploits a formal definition used for simulations to automatically generate a formal description of a parallel implementation. Gelenbe et al. [4] employs a more conservative formalism (queueing networks) in which the synchronization processes are not easy to describe. The SAN formalism, instead, was quite adequate to describe the behavior of a master-slave implementation. The definition of the events was very intuitive; their rates and probabilities were a little bit more hard to obtain but still intuitive. The stationary solution of the model provided enough prediction information about the behavior of the parallel version of the PEPS tool. The master and slaves nodes behaviors were clearly understood, and an analysis about the workload balance was also possible. The natural future work for this paper is to verify the accuracy of the proposed model by comparison with the real parallel implementation, in order to validate the correctness of the modelling choices made.

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